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Mortgage Durations and Price Moves

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Contents

This paper is an updated and modified version of *Effective and Empirical Durations of Mortgage Securities*, which was published by Salomon Brothers in 1996. I want to thank Hubert Chang for his assistance on the original paper. I also want to thank Vered Vaknin and Eileen Contrucci for preparation of the manuscript, and Franca Franz for her expert editing of the paper.

Introduction

Understanding the durations of mortgage-backed securities (MBSs) is critical for all participants in the MBS market. In this report, we present an analysis of both model- and market-based durations.

The first part of this report discusses model-based durations, which are derived from option-adjusted spread (OAS) models. We analyze the various assumptions that are part of standard *effective duration* calculations, and that typically lead to deviations between actual price moves and those predicted by effective durations. An expression for this deviation is derived in terms of the various *risk factors* that impact MBS prices and the *partial durations* of the MBS with respect to these risk factors in Appendix A. This expression is illustrated by a case study of Fannie Mae 6.5% prices over a volatile one-month period.

The second part of the paper discusses *empirical durations*, which are obtained by comparing actual MBS and Treasury price moves[1]. Empirical durations are popular sanity checks on model-based durations, but it is necessary to understand the characteristics and biases of these statistical estimators. The statistical properties of standard empirical duration estimates are derived in Appendix B, and this leads to a relationship between empirical and effective durations. This relationship can be used to intelligently combine the relevant information provided by empirical and effective durations, and leads to the concept of an *updated empirical duration.*

The final part of the report discusses hedging implications of using model-based and empirical durations.

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[1] For illustrative simplicity, our discussion is framed in terms of durations with respect to Treasuries. It goes without saying the concepts and results hold for other benchmarks, such as swaps.

Effective and Partial Durations

Recall that the effective duration is calculated is as follows:

1 For a given price P, calculate the OAS.

2 Shift the yield curve upwards in parallel by Δy and reprice the MBS at the original OAS. Call this price P^{+}.

3 Shift the yield curve downward by Δy in parallel and reprice the MBS at the original OAS. Call this price P^{-}.

4 Effective duration is then given by[2]

$$100 * \frac{(P^{-} - P^{+})}{P * 2 * \Delta y} \qquad \textbf{(1)}$$

An effective duration of 4.5, say, is often interpreted to mean that if, for example, rates decline by 100bp, the price of the MBS is projected to increase by approximately 4.5%. Price movements for other shifts are obtained through linear interpolation or extrapolation; for example, if rates increase by 20bp, the price of the MBS is projected to decrease by 0.2*4.5%, or 0.90%.

However, an examination of the effective duration calculation shows that a number of assumptions are embedded in the measure, notably the following:

- The yield curve moves in parallel;
- Volatilities remain unchanged as interest rates change;
- Mortgage rates change in parallel with Treasuries – in other words, current coupon mortgage spreads to Treasuries remain unchanged;
- The convexity of MBS prices is ignored;
- The OAS remains unchanged as interest rates change.

The price of an MBS depends on the whole yield curve, and many other variables (or *risk factors*), such as mortgage to Treasury spreads, volatilities, the OAS, and so on. The effective duration projects price moves *assuming a parallel yield curve shift and no change in other risk factors;* hence it can be interpreted as a measure of the price sensitivity of an MBS to a single risk factor, namely a parallel yield curve change. We can similarly calculate the MBS's price sensitivity to other risk factors. We define the *partial duration*[3] with respect to risk factor k as:

$$D_k = 100 * \frac{P(-\Delta k) - P(\Delta k)}{P * 2 * \Delta k} \qquad (2)$$

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[2] Equation (1) is a numerical approximation to the exact formula (-1/P)*(dP/dy).

[3] Partial Durations are also called *Key Rate Durations.* See *Beyond Duration: Risk Dimensions of Mortgage Securities,* Salomon Brothers, July 1992, and *Strategic Fixed-Income Investments,* Thomas Ho, Dow Jones-Irwin, 1990.

where $P(\Delta k)$ is the price of the MBS if risk factor k is changed by Δk and everything else is unchanged.

In Appendix A, we derive an expression for the change in price in terms of partial durations and changes in risk factors. Given the widespread use of effective duration to estimate likely MBS price changes, we will use the difference between the actual price move and the one projected by effective duration to discuss the various risk factors that influence mortgage price movements. Equation (3) gives a general formula for the difference between the actual price change and that projected by effective duration:[4]

$$\text{Actual} - \text{Projected Price Change} = \Delta P - \Delta \hat{P}$$

$$\cong P\left[-D_s \Delta s - D_v \Delta v - D_c \Delta c + \frac{1}{2} C_y \Delta y^2 - \sum D_{y_j} (\Delta y_j - \Delta y) \right] \qquad \textbf{(3)}$$

where D_k and C_k represent duration and convexity, respectively, with respect to risk factor k, and where s = OAS, v = volatility, c = current coupon spread, y = chosen Treasury yield (the ten-year), and y_j = key yield-curve rates. For each risk factor that affects the MBS price, the contribution to the price discrepancy is in essence the change in the factor times the partial duration of the MBS with respect to that factor.

Although the impact of departures from the effective duration assumption of a parallel yield curve assumption may be minor in many periods and, in fact, may average out over longer periods (unless there is a systematic dependence of model OASs on the level of rates), in a volatile market, with wide swings in Treasury yields, OASs, etc., caution needs to be exercised in using standard effective durations. We illustrate this latter point, and discuss the risk factors in Equation (3) with an example from a particularly volatile period, the Fall of 1998.

Deconstructing Mortgage Price Moves — A Case Study

On September 14, 1998, TBA Fannie Mae 6.5s were priced at 100-24, and had an effective duration of 3.1, while the ten-year Treasury yield was 4.86%. A month later (close of October 14, 1998), the ten-year yield had dropped 29bp. The effective duration would hence have implied a price move for the Fannie Mae 6.5s of:

-(100-24) (3.1) *(-.29)* = 0.90,*

or about 29 ticks. The actual price increase was 3 ticks, a discrepancy of 26 ticks. What led to this significant discrepancy? We decompose the price move using Equation (3) to calculate the contribution of the various risk factors.

Treasury Curve Reshaping. When using effective duration to predict price changes, the ten-year Treasury was used as a proxy for the whole curve; that is, in calculating

[4] See Appendix A for the derivation.

the projected price move of 0.90, we implicitly assumed that the whole yield curve declined in parallel by 29bp[5]. In fact, as shown in Figure 1, the curve did not move in parallel, but steepened over the month. Figure 1 also shows the partial durations of the Fannie Mae 6.5% with respect to different parts of the Treasury curve. We use four points to represent the whole curve. Note that the sum of the partial durations is approximately equal to the effective duration. This is by design — in shifting the yield curve around each of the four points, we ensure that the sum of the four shifts is a parallel shift.

Figure 1. Partial Durations (14 Sep 98) and Changes in Risk Factors for Fannie Mae 6.5s

	Treasury			
	2-Yr	5-Yr	10-Yr	30-Yr
Partial Durations	0.8	1.1	0.9	0.5
Change in Treasury Yield (Sep 14–Oct 14)	-65bp	-44bp	-29bp	-24bp

Source: Salomon Smith Barney.

Using Equation (3), the contribution of each maturity to the price discrepancy was:

2-Yr:	*-(100-24)(0.8)[-65bp-(-29bp)] =*	9 Ticks
5-Yr:	*-(100-24)(1.1)[-44bp-(-29bp)] =*	5
10-Yr:	*-(100-24)(0.9)[-29bp-(-29bp)] =*	0
30-Yr:	*-(100-24)(0.5)[-24bp-(-29bp)] =*	-1
		13 Ticks

Hence, *other things being equal,* the yield curve steepening would have led to an extra 13 ticks increase in the price of the 6.5%, versus what is implied by the effective duration and the change in the ten-year yield.

Current-Coupon Spread. The effective duration calculation assumes that the mortgage rates that are used to obtain prepayment projections move in parallel with Treasuries. In other words, it assumes that the spread between MBS current-coupon yields and Treasuries remains unchanged. In fact, the crisis in the financial markets in the Fall of 1998 led to a dramatic widening in spread products, including MBSs, and current coupon spreads widened 30bp over the month.

The current-coupon spread duration measures the impact on the MBS price of a change in this spread. In our calculations, it is calculated for a 10bp change in the spread. For the Fannie Mae 6.5%, the current spread duration was -0.13 at the beginning of the period[6], which, from Equation (3), means that the contribution to the discrepancy was

$$-(100-24)(-0.13)(30bp/10bp) = 12 \text{ ticks}$$

The widening in current coupon spreads helps the MBS, as it implies higher mortgage rates and hence a lower degree of refinancing risk.

[5] In the notation used in Equation (3), the *y* is the 10-year yield.

[6] A widening in current-coupon spreads raises mortgage rates, which reduces refinancings and hence helps MBSs, giving a negative partial duration. As indicated in the text, this duration is calculated assuming a 10bp change in the current-coupon spread.

Volatilities. Volatilities increased over the period, with, for example, the 1x10 swaption volatility going up 3.63% and the 5x10 swaption volatility going up 0.63%. For simplicity, we will use just these two instruments to capture the impact of volatility changes, since in our model they contribute a large fraction to the total volatility impact on the 6.5s.

The *vol duration* of the 6.5% is 0.08 with respect to the 1x10 swaption, and 0.12 with respect to the 5x1 swaption (the positive durations reflect the adverse impact of an increase in volatility on MBS prices). Hence, from Equation (3), the contributions of the higher volatilities to the price change are:

1x10:	*-(100-24)(.08)(3.63%) =*	*-9.5* Ticks
5x10:	*-(100-24)(0.12)(0.63%) =*	*-2.5*

or -12 ticks in total. Thus, the increase in vols means a 12-tick drop in the price of the Fannie Mae 6.5%, other things being equal.[7]

Convexity or Asymmetric Price Movements. Effective duration is, in essence, obtained by averaging projected price changes when rates move up and down. If price changes are asymmetric, then effective duration will tend to overproject or underproject the changes. In many cases, MBSs have negative convexity,[8] which means that, other things being equal, effective duration will over-project price increases when rates move down and underproject price declines when rates move up.

Equation (3) has a term involving the convexity. Since this term does not have the same form as the others in Equation (3), we give a brief derivation. If we define D^+ and D^- to be the durations when rates move up and when rates move down, respectively, then

$$\text{Effective Duration} = 0.5*(D^- + D^+)$$

Hence, if with the benefit of hindsight we knew that rates were going to move down, we would use D^- for projecting the price move. The difference versus using effective duration is

$$D^- - 0.5*(D^- + D^+) = 0.5*(D^- - D^+)$$
$$\cong 0.5*\Delta y * \mathit{Convexity}$$

Hence, the difference in projected prices is approximately

$$\text{Price}*\Delta y*(0.5*\Delta y* \mathit{Convexity}) = \text{Price}*0.5*(\Delta y)^2 *\mathit{Convexity},$$

which is the term given in Equation (3)

The Fannie Mae 6.5% had a convexity of -3.2 at the beginning of the period, which implies a negative impact on the projected price appreciation as rates rallied, equal to

$$(100\text{-}24)*0.5*(-29bp)^2 *(-3.2) = -4 \text{ ticks}$$

[7] Note that the vols are making an explicit contribution to the price change, since we are using market vols. If we were using fixed vols, the impact of vols would show up as part of the change in OAS.

[8] For a discussion of MBS convexity and how it is calculated, see *Guide to Mortgage Securities*, Lakhbir Hayre, Salomon Smith Barney, May 1999.

OAS. Changes in the OAS reflect changes in risk factors other than the ones (discussed above) which are explicitly accounted for in the OAS calculations; for example, concerns about supply, hedge fund liquidations, changes in prepayment views, and so on as well as any general widening in spread product (assuming we are calculating OASs to Treasuries). This last point has been especially relevant over the last two years, as the correlation between movements in spread products and Treasuries has weakened, leading to increased volatility in OASs calculated to the Treasury curve (MBS OASs to swaps have been much more stable).

The OAS of the Fannie Mae 6.5% widened 27bp over the period (to market vols). The *spread duration* of the 6.5s was 4.1 at the beginning of the period, so the price impact of the OAS widening was

$$-(100\text{-}24)(4.1)(27bp) = -35 \text{ ticks}$$

Net Impact on Price Change. Summarizing the analysis above, the various risk factors had the following impact on the price of the Fannie 6.5%:

Treasury Yield Curve Reshaping:	+13 Ticks
Increase in Current Coupon Spread:	+12 Ticks
Increase in Volatilities:	- 12 Ticks
Convexity:	- 4 Ticks
Widening in OAS:	- 35 Ticks

The sum of these price changes comes to [13+12 -12 -4 -35], or -26 ticks, which is about the same as the difference between the actual price change and the change implied by effective duration. In other words, the risk factors discussed above explain almost all the deviation between the actual price change and that projected by effective duration. This is to be expected, since as discussed above, the final risk factor, the OAS, incorporates the effect of risk factors not explicitly discussed here.

Time Value, or the Cost of Carry

One risk factor that is not included in the above discussion, but that can sometimes explain part of the discrepancy between actual and duration-projected price moves, is the difference in *carry adjustments* between the two dates. This term refers to the change in prices that occurs as we move closer to the settlement date, reflecting the difference between the yield on the bond and short-term money market rates. For example, MBS trading is typically for forward settlement, and prices tend to increase as the settlement date approaches (and hence the higher yield of the MBS, versus cash, will be obtained sooner). This increase in price will not be reflected in effective durations, and hence in the price change projections based on these measures.

For periods of a few days or less, the time factor will usually not be important. Even for longer periods, the impact of time typically depends not on the difference between the two dates but on the time from each date to the next settlement date. Because of these considerations, we have not explicitly included time as one of the risk factors in Appendix A, but it is something investors should keep in mind. Using carry-adjusted prices will typically remove most of the effect of time.

Empirical Durations

Empirical durations refer to estimates of MBS price elasticity, typically with respect to Treasury rates, obtained from market data. While there are many possible ways of obtaining such measures, the standard approach involves regressing percentage MBS price changes against corresponding Treasury yield changes. We describe this method in more detail below, discuss what information it provides, and derive a relationship between empirical and effective durations. This relationship is used to derive an "updated" empirical duration, which combines the effective duration with the pertinent information provided by the empirical duration. Also discussed are alternative methods of calculating empirical durations, including those based on a fixed relative coupon (or constant dollar price). However, as we highlight, these other measures have their own limitations.[9]

Standard Empirical Duration Estimates

The usual method for calculating empirical durations is to regress daily MBS percentage price changes against corresponding yield changes for a benchmark Treasury (typically the ten-year). If P denotes MBS price and y the Treasury yield, then by definition,

$$dP/P = -\text{Duration} * dy \qquad \textbf{(4)}$$

If $\Delta P/P$ and Δy are the actual price and yield changes on a given day, then based on Equation (4) we assume that

$$\Delta P/P = \alpha - \beta * \Delta y + \text{noise term} \qquad \textbf{(5)}$$

where β is the "true" duration, and α is a constant term. Given data ($\Delta P/P$, Δy) for a number of days, standard regression methods can be used to obtain an estimate for β (see Equation (B2) in Appendix B). This estimate, $\hat{\beta}$, say, is taken to be the empirical duration for the period.[10]

The Relationship Between Empirical and Effective Durations

Earlier in this report, we pointed out that the price of an MBS will depend on a number of factors: various points on the yield curve, volatilities, the OAS, and so on. Appendix B derives an expression for the empirical duration estimate β obtained

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[9] Earlier work on empirical durations includes papers by Pinkus & Chandoha, (*Journal of Portfolio Management*, Summer 1986), DeRosa, Goodman and Zazzarino, (*Journal of Portfolio Management*, Winter 1993), and Breeden (*Journal of Fixed Income*, September, 1991 and December, 1994). The focus of these papers is measuring market durations and (in the Breeden papers) on their hedging effectiveness, whereas ours is on exploring the theoretical relationships between empirical and effective durations.

[10] Why is an intercept term used in Equation (5)? In other words, why not use

$$\Delta P/P = -\beta * \Delta y + \text{noise term}$$

to estimate the duration? The reason is that having an intercept term "detrends" the data, so that the estimate for β is not distorted through having to incorporate price changes unrelated to yield changes. In practical terms, it typically makes little difference as to whether an intercept term is used or not.

using Equation (2) in terms of the true duration β and these various risk factors. If s denotes OAS, v denotes volatility,[11] and so on, then as shown in Appendix B,

$$\hat{\beta} \cong \beta + \mu + D_s * Corr(\Delta s,\Delta y)*Vol(\Delta s)/Vol(\Delta y) + D_v * Corr(\Delta v,\Delta y)*Vol(\Delta v)/Vol(\Delta y) + \ldots\ldots \quad \textbf{(6)}$$

where $\hat{\beta}$ = Empirical Duration Estimate

β = Current Effective Duration

μ = average difference between current effective duration and the effective durations over time period used for the data

D_k = Duration of MBS with respect to risk factor k

$Corr(\Delta k, \Delta y)$ = sample correlation between changes in risk factor k and changes in y over the sample time period

$Vol(U)$ = sample standard deviation (or volatility) of daily changes in variable U over the sample time period.

In practice, the most important factor is a change in OAS. *If we ignore other risk factors, ignore the effect of noise, and assume that the duration is fairly stable over the time period used,* then, approximately,

$$\text{Emp Dur Estimate} = \hat{\beta} \cong \beta + D_s * Corr(\Delta s, \Delta y) * Vol(\Delta s)/Vol(\Delta y) \quad \textbf{(7)}$$

where D_s is the OAS duration of the MBS, Corr (Δs, Δy) is the correlation between OAS and yield changes over the time period used, and Vol (Δs) and Vol (Δy) denotes the standard deviation of Δs and Δy respectively, over the sample time period.

Why Effective Durations are Often Longer than Empiricals. Equation (7) states that the difference between empirical and effective durations is proportional to the correlation between daily OAS and Treasury yield changes. If there is significant directionality between daily OAS and yield changes, with a negative correlation between them (so that a drop in yield leads to widening in OAS), then the empirical duration will be shorter than the effective duration. **This will be true even if there is no net change in OAS over the period, and the cumulative price change is in line with that predicted by effective duration.**

Empirical and effective durations will tend to diverge when there is a high correlation between OAS and yield changes, which tends to occur during periods when there is a high degree of prepayment fears.

[11] For ease of notation, we will assume just one volatility, although our formulation allows us to include as many volatilities (and other risk factors) as desired.

Combining Empirical and Effective Durations. Investors who lean towards empirical durations should instead use an adjusted version derived from **Equation (6).**

We define this as

$$\text{Updated Emp Dur} = \text{Emp Dur} - \mu \cong \beta + D_s \,{*}\mathrm{Corr}(\Delta s,\Delta y){*}\mathrm{Vol}(\Delta s)/\mathrm{Vol}(\Delta y) + D_v \,{*}\mathrm{Corr}(\Delta v,\Delta y){*}\mathrm{Vol}(\Delta v)/\mathrm{Vol}(\Delta y) + \ldots \quad \textbf{(8)}$$

This is, in effect, equivalent to the empirical duration adjusted for duration changes over the sample time period. In other words, the updated empirical duration incorporates the information provided by empirical duration and also uses current market information, as captured by the effective duration. **It can alternatively be thought of as the effective duration adjusted for the correlations between changes in the yield and changes in risk factors displayed by recent market data.**

The updated empirical duration is generally very close to the empirical. Any differences between the two reflect the effect of recent market moves that can make the empirical durations out of date. For example, in the Spring of 1995, when rates were falling, the updated empirical duration declined faster. Similarly, a year later, when rates were rising, the updated empirical duration rose faster.

Constant Relative Coupon (or Constant Price) Durations. MBS durations change with interest rates, so that if rates have moved substantially, the empirical duration for a given coupon can be a poor indicator of the likely duration going forward. This had led to the development of empirical durations for a fixed relative coupon (or, more or less equivalently, for a fixed dollar price), where we estimate the empirical duration not for a fixed coupon (say 7.5s), but a fixed relative coupon (for example, the current coupon). Thus, the price moves used in the calculation may not (and typically will not) be for the same MBS over the whole time period. For example, if we are calculating the empirical duration for the current coupon, then for each day, the price move will be for the MBS that was the current coupon on that particular day.

Though empirical durations by relative coupon can provide valuable information, there can be problems with this solution to a real problem (durations changing over the time). The first and obvious one is that different MBSs may differ in key features such as WAMs, previous prepayment history, etc., and therefore will not display the same durations even when they are the same relative coupon. Second, as the last several years have made clear, prepayments, and hence durations, depend not just on the relative coupon but also on the absolute level of rates. Thus, even for the same relative coupon, durations can change substantially over time.

Even for discount relative coupons, the duration can change by more than half a year in a single month, and for a cuspy coupon (such as current coupon plus 200bp), the duration has sometimes changed by a factor of two or more in a single month.

A practical problem with relative coupon durations is that available data may be suspect or may not even exist. For example, in the Spring of 1995, after rates started falling sharply, the “+200bp” durations were based on price moves of 10s and higher coupons, which tend to be illiquid.

Empirical Durations Based on Price Levels, Not Price Changes

An implicit assumption in empirical duration calculations is that the Treasury yield change, on a given day, impacts the MBS price that same day. This is what is expressed, for example, by Equation (5) above. While this is a reasonable assumption for liquid actively traded securities, it may not be true in other cases.

An example is provided by high premium pass-throughs. The float on these MBSs is small, much of the trading is on a specified pool basis and, therefore, there is not much of a TBA market. As a result, prices for high-premium TBAs often react with a lag, responding cumulatively to several days worth of Treasury curve changes. Hence, comparing daily price changes with corresponding Treasury yield changes suggests very little relationship, leading to a low estimate for empirical duration.

An alternative approach, described in an earlier article,[12] is to compare price levels with yield levels, i.e. rather than regressing $\Delta P/P$ versus Δy — we can, for example, regress log P versus y.[13] The negative of the slope will be the empirical duration estimate.

It is important to be clear as to what this empirical duration estimate measures. It describes the relationship between MBS price and Treasury yield *levels* over a period of time, rather than the relationship between day-to-day *changes*. As a result, it may lead to a poor hedge against daily yield curve fluctuations. Even for it to be useful in deriving a long-term hedge, it has to be assumed that changes in OAS, etc., that occurred during the historical period would be repeated over the time period for the hedge.

Partial Empirical Durations. Partial durations can be used to hedge against yield curve reshaping. We can estimate empirical partial durations by using a multiple regression version of **Equation (5)**:

$$\Delta P/P = \beta_0 - \beta_1 {*} \Delta y_1 - \ldots - \beta_j \Delta y_j + \text{noise term}$$

for selected Treasury yields $y_1, \ldots, y_j$. However, Treasury yields of different maturities tend to be highly correlated, leading to regression estimates for the b's that can be unstable. As a result, partial empirical durations do not seem to be widely used in the market.

[12] See *Bond Mortgage Roundup: Strategy*, November 3, 1995, Salomon Brothers Inc.

[13] Log P versus y is preferable to P versus y, since if log P = a + by, then [dP/dy]/P = -Duration = b.

Durations and Hedging

Traditionally, either effective or empirical duration has been used to calculate hedge ratios for MBSs, typically with respect to the ten-year Treasury. As we have discussed in this report, the choice between using empirical or effective durations for hedging involves making fundamentally different assumptions about the relationship between past and future price movements. To reiterate our findings (note that Equation (A9) in Appendix A gives the difference between predicted and actual price changes if we use effective duration, while Equation (B7) in Appendix B gives the corresponding difference if we use empirical duration):

Effective duration hedges against a parallel yield curve shift, and assumes that other risk factors are unchanged. If effective duration is used, the prediction error will be due to changes in risk factors such as yield curve reshaping, volatilities, current-coupon spreads, OASs, and to second order (convexity) effects.

Empirical duration assumes that past relationships (such as correlations) between changes in the ten-year Treasury yield[14] and changes in risk factors such as OASs, volatilities, etc. will hold going forward. In other words, for a given yield change, the OAS and other factors will change by amounts implied by past patterns (where past means the time period over which the empirical duration is calculated). Therefore, hedging errors will be due to OASs, etc. changing by amounts different than those implied by past data. In addition, hedging errors may arise due to the duration changing over the sample time period and due to noise.

Thus, effective duration is preferable to empirical duration if we believe that, for example, correlations between OAS and yield changes do not generally show any systematic pattern (i.e., our OAS model does not, on average, display any rate dependence) and that we should not try to predict such correlations. Even if daily OAS and yield changes do show a correlation, changes over a week or a month may not and, as a result, effective duration may still be better unless reducing day-to-day fluctuations in our position is of critical importance. Conversely, the empirical durations will be preferable if we believe that past correlations between Treasury yield changes and the various risk factors will repeat themselves going forward (e.g., if the OASs from our model show a systematic and predictable rate dependence). However, in this latter case, it is preferable to use the updated empirical duration-defined next, to take account of substantial market moves over the time period used for empirical duration calculations, and to eliminate the effect of noise.

Note that neither effective nor empirical durations will lead to a hedge against price moves that are *uncorrelated* with yield moves. For example, if the OAS widens or tightens in a way unrelated to Treasury yield changes – due to, for example, general spread movements of spread products versus Treasuries – then Treasuries will not provide a hedge for the MBS price change resulting from the OAS move.

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[14] Or, more generally, in the benchmark rate against which empirical durations are calculated.

Hedging Implications

It is self-evident that we cannot predict changes in the various risk factors that influence MBS prices. Therefore, if we want to minimize short-term fluctuations in a hedged position, we should use multiple instruments to hedge MBSs against movements in the various risk factors, using partial durations to compute hedge ratios. Although some of this is straightforward (such as using several Treasuries to hedge against yield curve reshaping, or using options to hedge volatility changes), the difficulty is likely to lie in hedging the residual risk, as incorporated in OAS changes. OAS changes can be decomposed into those due to general widening in spread product, and those specific to MBSs, such as prepayment or supply concerns or other technicals. We can attempt to hedge the first component by, for example, using **swaps** rather than Treasuries; for example, in the case study of the Fannie Mae 6.5s analyzed earlier in this chapter, over the month, the OAS of the 6.5% versus swaps widened only 9bp, versus the 27bp against on-the-run Treasuries.[15] Hedging against the second component is more difficult and specific to the security, although the IO market does provide a means of hedging against changes in market expectations of prepayments.

If short-term fluctuations are not a major concern, and we have a long-term horizon, then using the effective duration may suffice. The basic assumption is that changes in risk factors will average out over time. Some evidence that this will occur is provided by the fact that OASs have not shown a systematic downward or upward trend over time, implying that changes in the various risk factors generally reverse themselves over time.

[15] Off-the-runs Treasuries would have also been better than the on-the-runs - the OAS to the Treasury Model curve widened 14bp versus the 27bp for the on-the-runs.

Appendix A. Price Changes and Effective Durations

Actual Price Move

Let k_1 through k_N be risk factors such as OAS, volatilities, yield-curve rates, etc. For given changes Δk_1 . . ., Δk_N in these risk factors, using a Taylor Series expansion, a mortgage security's price change can be expressed as

$$\Delta P = \sum_{j=1}^{N} \left[\frac{\partial P}{\partial k_j} \Delta k_{j+} \frac{1}{2} \frac{\partial^2 P}{\partial k_j^{\,2}} \Delta k_j^{\,2} \right] + \text{cross/higher order terms}$$

Dividing by the original price P, the percentage change in price is given by

$$\frac{\Delta P}{P} = \frac{1}{P} \sum_{j=1}^{N} \left[\frac{\partial P}{\partial k_j} \Delta k_j + \frac{1}{2} \frac{\partial^2 P}{\partial k_j^{\,2}} \Delta k_j^{\,2} \right] + \text{cross/higher order terms} \qquad \textbf{(A1)}$$

Defining partial duration with respect to *k* as $D_k = -\frac{1}{P}\frac{\partial P}{\partial k}$, and partial convexity with respect to k as

$C_k = \frac{1}{P}\frac{\partial^2 P}{\partial k^2}$, then **(A1)** becomes

$$\frac{\Delta P}{P} = \sum_{j=1}^{N} \left[-D_{k_j} \Delta k_j + \frac{1}{2} C_{k_j} \Delta k_j^{\,2} \right] + \text{cross/higher order terms}$$

Limiting risks to OAS, single volatility, current coupon spread, and yield-curve risks,[16] and neglecting all higher-order terms except yield-curve convexity gives

$$\frac{\Delta P}{P} = -D_s \Delta s - D_v \Delta v - D_c \Delta c - \sum D_{y_j} \Delta y_j + \frac{1}{2} \sum C_{y_j} \Delta y_j^{\,2} \qquad \textbf{(A2)}$$

where *s* = OAS, *v* = volatility, c = current coupon spread, and y_j = key yield-curve rates.

For a given yield curve rate, y, say let D_y and C_y be the effective duration and convexity. Note that, neglecting higher-order terms, $D_y = \sum D_{y_j}$ and $C_y = \sum C_{y_j}$. We can now rewrite **(A2)** as approximately

$$\frac{\Delta P}{P} = -D_s \Delta s - D_v \Delta v - D_c \Delta c - D_y \Delta y + \frac{1}{2} C_y \Delta y^2 - \sum D_{y_j} \left(\Delta y_j - \Delta y \right) \qquad \textbf{(A3)}$$

[16] This is by no means a complete set of risk factors or durations; among others could be prepayment and time durations. The risk factors cited are generally the most important for typical MBSs, and in addition (apart from OAS) are observable.

where we have ignored terms involving $\left(\Delta y_j{}^2 - \Delta y^2\right)$. Note that the $\left(\Delta y_j - \Delta y\right) = \Delta\left(y_j - y\right)$ terms measures yield-curve reshaping.

Effective Duration

Effective duration assumes that the yield curve shifts in parallel and other risk factors are unchanged. That is, $\Delta y_j \equiv \Delta y, \Delta s = 0, \Delta c = 0,$ and $\Delta v = 0.$ Equation **(A3)** then becomes (ignoring higher-order terms)

$$\frac{\Delta P}{P} \cong -D_y \Delta y + \frac{1}{2} C_y \Delta y^2 \qquad \textbf{(A4)}$$

Assume that $\Delta y > 0$. Then if rates backup by , (A4) gives

$$\frac{P^{+} - P}{P} \cong -D_y \Delta y + \frac{1}{2} C_y \Delta y^2 \qquad \textbf{(A5)}$$

Similarly, if rates rally by Δy, (A4) gives

$$\frac{P^{-} - P}{P} \cong D_y \Delta y + \frac{1}{2} C_y \Delta y^2 \qquad \textbf{(A6)}$$

(A6) - (A5) gives

$$\frac{P^{-} - P^{+}}{P} \cong 2 D_y \Delta y$$

Effective duration is then given by

$$\left[\frac{P \; - P^{+}}{P}\right]\left[\frac{1}{2\Delta y}\right] \cong D_y = -\frac{1}{P}\frac{dP}{dy} \qquad \textbf{(A7)}$$

Hence, for a given yield change Δy, the projected percentage change in price using effective duration is given by

$$\frac{\Delta \hat{P}}{P} = -(\text{Effective Duration}) \; \Delta y \; \cong D_y \Delta y \qquad \textbf{(A8)}$$

Difference Between Actual and Projected Price Changes

The difference between the actual percentage price change and that projected by effective duration is given approximately by **(A3) - (A8).**

$$\frac{\Delta P}{P} - \frac{\Delta \hat{P}}{P} \cong -D_s \, \Delta s - D_v \, Dv - D_c \, \Delta c + \frac{1}{2} C_y \, \Delta y^2 - \sum D_{y_j} \left(\Delta y_j - \Delta y\right) \qquad \textbf{(A9)}$$

Appendix B. Linear Regression Estimates of Duration

Linear Regression

Empirical durations are typically calculated using a linear regression model given by

$$Y_t = \alpha - \beta X_t + \varepsilon_t \qquad \textbf{(B1)}$$

where $Y_t = \dfrac{\Delta P}{P} = \dfrac{P_t - P_{t-1}}{P_{t-1}}$, or the proportional change in price, and

$X_t = \Delta y_t = y_t - y_{t-1}$, or the daily yield change, e.g. for the 10-year Treasury.[17]

Least squares minimization leads to a slope estimator of

$$\hat{\beta} = -\frac{\sum Y_t X_t - \frac{1}{N}\sum Y_t \sum X_t}{\sum X_t^{\,2} - \frac{1}{N}\left(\sum X_t\right)^2} \qquad \textbf{(B2)}$$

where N = number of observations. The empirical duration is then taken to be $\hat{\beta}$.

What Does Empirical Duration Measure?

Equation **(B1)** assumes that α and β are constant. In fact, referring to Equation **(A3)** in Appendix A and neglecting higher-order terms,

$$\frac{\Delta P}{P} \cong \left[-D_s\,\Delta s - D_v\,\Delta v - D_c\,\Delta c + \frac{1}{2} C_y\,\Delta y^2 - \sum D_{y_j}\left(\Delta y_j - \Delta y\right)\right] - D_y\,\Delta y \qquad \textbf{(B3)}$$

For day *t*, let α_t be the value of the term in brackets, and let $\beta_t = D_y$.

Hence, the true relationship is

$$Y_t = \alpha_t - \beta_t X_t + \varepsilon_t,$$

where ε_t captures influences on $\dfrac{\Delta P}{P}$ other than those shown in Equation (B3).

[17] Note a change in notation: in Appendix A, Δy_k denoted the change in the k^{th} yield-curve rate, whereas here Δy_t represents the change in a given yield-curve rate between times t and (t-1).

Substituting this Y_t into Equation **(B2)**, the numerator becomes

$$\sum Y_t X_t - \frac{1}{N}\sum Y_t \sum X_t$$

$$= \sum \left(\alpha_t - \beta_t \Delta y_t + \varepsilon_t\right)\Delta y_t - \frac{1}{N}\sum \left(\alpha_t - \beta_t \Delta y_t + \varepsilon_t\right)\sum \Delta_{y_t}$$

$$= \sum \alpha_t \Delta y_t - \sum \beta_t (\Delta y_t)^2 + \sum \varepsilon_t \Delta y_t - \frac{1}{N}\sum \alpha_t \sum \Delta y_t + \frac{1}{N}\sum \beta_t \Delta y_t - \frac{1}{N}\sum \varepsilon_t \sum \Delta y_t$$

Let β denote the current value of β_t (i.e. of $-\frac{1}{P}\frac{dP}{dy}$), and define $\mu_t = \beta_t - \beta$.

Without loss of generality, we can assume that the ε_t term is noise. We further assume that μ_t and Δy_t have a very low correlation.

In addition, if we define

$$\text{Sample Covariance} = \mathrm{COV}(A,B) = \frac{1}{N}\left[\sum (A_i B_i) - \frac{1}{N}\sum A_i \sum B_i\right]$$

$$\text{Sample Variance} = \mathrm{VAR}(A) = \frac{1}{N}\left[\sum A_i^{\,2} - \frac{1}{N}\left(\sum A_i\right)^2\right]$$

then straightforward algebra shows that, approximately

$$\hat{\beta} = \beta + \mu - \frac{\mathrm{COV}(\alpha, \Delta y)}{\mathrm{VAR}(\Delta y)} + \mathrm{NOISE} \tag{B4}$$

where μ is the average of μ_t over the sample period (that is, it is the average difference between the current effective duration and the ones from the data period), and *NOISE* refers to the terms involving ε_t.

From Equation **(B3)**, $\alpha_t = -D_s \Delta s_t - D_v v_t - D_c \Delta c_t + \frac{1}{2} C_y (\Delta y_t)^2 + \ldots$

Now, for any variables U and V,

$$\frac{\mathrm{COV}(U,V)}{\mathrm{VAR}(V)} = \rho_{uv}\frac{\sigma_u}{\sigma_v},$$

where ρ_{uv} = correlation between U and V

σ_u = standard deviation of U

σ_v = standard deviation of V

Equation **(B4)** can now be written as

$$\hat{\beta} \cong \beta + \mu + D_s \rho_{\Delta s \Delta y} \frac{\sigma_{\Delta s}}{\sigma_{\Delta y}} + D_v \rho_{\Delta v \Delta y} \frac{\sigma_{\Delta v}}{\sigma_{\Delta y}} + \ldots + \text{NOISE} \qquad \textbf{(B5)}$$

where $\rho_{\Delta s \Delta y}$, etc. are sample correlations.[18]

Prediction Error Using Empirical Duration

Equation **(A9)** in Appendix A gave the difference between actual and projected prices using effective duration. The corresponding error using empirical duration is

$$\frac{\Delta P}{P} - (-\hat{\beta}\Delta y) \cong \mu\Delta y + D_s \left[\rho_{\Delta s \Delta y} \frac{\sigma_{\Delta s}}{\sigma \Delta_y} \Delta y - \Delta s \right] + D_v \left[\rho_{\Delta v \Delta y} \frac{\sigma_{\Delta v}}{\sigma_{\Delta y}} \Delta y - \Delta v \right] + \ldots \qquad \textbf{(B6)}$$

Interpretation of Prediction Error

Equation **(B6)** is easier to interpret if we first note that the linear regression predicted value for, say, Δs, based on the sample data is $\rho_{\Delta s \Delta y} \frac{\sigma_{\Delta s}}{\sigma_{\Delta y}} \Delta y$.

In other words, if we had to predict the change in Δs given Δy, then historical data would give the linear regression predictor as

$$\hat{\Delta} s = \rho_{\Delta s \Delta y} \frac{\sigma_{\Delta s}}{\sigma_{\Delta y}} \Delta y$$

Hence the prediction (or hedging) error shown in Equation **(B6)** can be rewritten as

$$\text{Error} \cong \mu\Delta y + D_s (\hat{\Delta} s - \Delta s) + D_v (\hat{\Delta} v - \Delta v) + \ldots \qquad \textbf{(B7)}$$

Equation **(B7)** states that if we use empirical duration, then hedging errors will be due to changes in OAS, and other risk factors displaying correlations with changes in *y* and relative volatilities that differ from those displayed in the past. In contrast, Equation **(A9)** in Appendix A states that if we use effective duration, then any differences between actual and projected prices will be due to *changes* in OAS, volatilities, and other risk factors.

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[18] For simplicity, we have assumed that D_S, D_V, ... are relatively constant over the sample period. In general, under fairly reasonable assumptions, Eq. (B5) will hold with D_S, ... replaced by sample averages.

ADDITIONAL INFORMATION AVAILABLE UPON REQUEST

(7024N, 7008N, 7224N, 7014N)

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