Optimal Put Exercise: An Empirical Examination of Conditions for Mortgage Foreclosure

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Abstract

Implicit in option-pricing models of mortgage valuation are threshold levels of put-option value that must be crossed to induce borrower default. There has been little research into what these threshold values are that come out of pricing models, or how they compare to exercised option values seen in empirical data. This study decomposes boundary conditions for optimal default exercise to look at the economic dynamics that should lead to optimal default timing. Empirical data on FHA insured mortgage foreclosures is then examined to discern the predictive influence of optimal-option-valuation-and-exercise variables on observed default timing and values. Interesting results include a new understanding of how to measure and use property equity variables during economic downturns, house price index ranges over which default is exercised for various classes of borrowers, and implied differences in appreciation rates between market price indices and foreclosed properties.

I. Introduction: Ruthlessness of Default-Option Exercise

The contingent-claim approach to modeling mortgage default has led to a lengthy debate over the ruthlessness of borrowers in exercising implicit put options imbedded in mortgage contracts. Original option pricing models described ruthless or frictionless default as borrowers giving up property rights in exchange for release from the mortgage obligation whenever the market value of the mortgage exceeds the value of the underlying property. The idea of ruthless default arose from early mortgage pricing models where boundary conditions for default were set at the point where the market value of the mortgage equaled the property value (see, Titman and Torous, 1989).

Are there significant levels of transaction costs that impede frictionless option exercise, or is the degree of non-ruthlessness observed in practice a function of the value of maintaining the option to default in the future? Debate on this question has spawned numerous journal articles and conference debates. Option-pricing theorists have adopted the valuation of keeping open the option to default in the future as a way of defining default boundaries that can vary substantially from the point where equity is zero (Kau, et al, 1992). This is simply a way of saying that borrowers will not default until they can extract as much value as possible from the put option. Other authors have questioned the pure options approach to mortgage pricing and have instead promoted a borrower-solvency paradigm, arguing that there are empirical transaction costs to default which are substantial and which make for less-than-ruthless default exercise as the norm (Quigley and Van Order, 1995; Vandell, 1995; Elmer, 1997, and Deng, Quigley and Van Order, 2000).

Ruthlessness is a measure of the *value* of default, given default occurs. There have been few empirical studies of this important aspect of mortgage pricing. Can pure option-pricing approaches to default modeling capture a realistic distribution of exercised default-option values? And, if so, what kinds of model parameters are needed to assure realism? Kau and Keenan (1999) show the distribution of simulated default outcome ("severity") levels that arise from their 1993 option-pricing model of default. They claim realism in that mean rates from their simulation resemble mean empirical rates shown by Lekkas, et al (1993). Kau and Keenan therefore claim that a pricing model built on a default boundary that captures the value of future default can produce reasonable results. However, following the Kau and Keenan (1993) study of default rates, the Kau and Keenan (1999) severity analysis includes an arbitrary distribution of non-optimal (trigger event) defaults which takes the form of a Poisson process with shocks of 50% PSA arriving at payment dates. Thus, the Kau-Keenan model is a hybrid: it is not a pure option pricing model. Ambrose and Capone (1998) also adopt a hybrid approach by classifying defaulting borrowers into two classes: put-option exercisers and trigger-event defaulters.

The Kau-Keenan (1999) study does not answer the question of how realistic are exercised default-option values produced by a pure option pricing model but, rather, it suggests that the option-pricing paradigm can be modified and still allow for some level of optimal (wealth maximizing) default. Non-optimal default is defined as borrowers defaulting before it is optimal because of a forced mobility. The authors conclude that they hope their work will help direct empirical research. Empirical analysis both of default rates and values is required to understand to what extent borrowers think of default as a financial option (and a legal right in the mortgage contract) versus the unintended consequence of insolvency (trigger events).

In this study we focus on default-option value and its impact on foreclosure of FHA-insured mortgages. Our work provides a look into ruthlessness in put-option exercise by testing sensitivity of default timing and value to the underlying economics that signify that default boundary conditions are being crossed. Yet we allow “ruthlessness” to encompass the value of delayed option exercise. By focusing on the analytics of crossing boundary thresholds, we better understand the effects of property market cycles on optimal-default decisions, and how to specify variables to use in empirical/statistical models of default incidence and (option) value.

In section II of this paper we review the existing literature on default-option value and default severity. That literature looks at the issue from the lender or investor’s perspective, hence the variable of interest is default *loss severity* rather than default *option value*. There have been few studies, and there is no standard definition of the severity rate measure used. In section III we decompose the theoretical default-option boundary into property value and interest rate effects, in section IV we develop empirical variables for statistical analysis that follow that analysis, and in section V we present a two-stage estimation procedure, results, and simulations of exercised default values. Conclusions are provided in section VI.

II. Previous Analyses of Default-Option Value

The default (put) option is optimally exercised when current negative property equity (plus call option value) outweighs the expected value of default in the future. Default loss severity is a byproduct of default exercise (optimal or non-optimal). A small number of empirical studies attempt to model actual severities to see what factors may be linked to their size.[1] In general, these studies entail regressions of severity rates on original loan-to-value and loan age, without any attention to developing option-value variables. These studies also do not provide frequency distributions of observed severity rates, so the reader does not know what put-option values actually trigger default in these models.

The Office of Federal Housing Enterprise Oversight recently published a proposed regulation that includes a loss severity model.[2] Severity includes all costs to the lender, but the centerpiece of the analysis is a regression of equity loss found on foreclosed properties. The actual negative property equity of foreclosed-properties (mortgage balance less final property value) is regressed on what property equity would be under (market) average rates of house price appreciation. Through some statistical transformations, the regression predicts average equity loss as a function of the first and second moments of the area house-price distribution.[3] While this analysis relates default-option value to housing market conditions, it does not address the question of whether borrowers are acting in accordance with option theory.

[1] See for example, Clauretie (1990), Lekkas, Quigley, and Van Order (1993), Crawford and Rosenblatt (1995), and Berkovec, et al (1998).
[2] Office of Federal Housing Enterprise Oversight (1999), p. 18188f.
[3] The log of one plus the equity loss is regressed on the normalized (or standardized) distance between a house price index and an index of the actual loan balance. The normalization occurs by dividing the distance measure by a measure of the standard deviation of house price growth rates in the region.

III. Default-option value and the Decision to Default

A. Defining Default

In this study we concentrate on default-option value in foreclosure. This is most appropriate for testing option-pricing theories because foreclosure is a straight put option exercise. Over the past ten years, the mortgage industry has grown to understand default-option value more completely, and has moved toward proactive management of delinquencies and steering defaulting borrowers to non-foreclosure resolutions. Such alternative resolutions are designed to lower lender cost (severity) from mortgage default. Borrowers may or may not receive the same default-option value under an alternative to foreclosure, depending on the actual workout option and the leverage of the lender to push some of the costs of default back onto the borrower.

Ambrose and Capone (1996) discuss lender motivations to attempt foreclosure alternatives, based on a probabilistic analysis of resolution options. Capone (1996) provides a history of industry management of default and foreclosure, along with an analysis of the legal parameters within which mortgage default takes place.

In spite of this change in industry direction, mortgage pricing models in the literature continue to rely on the historic paradigm of defaults equaling foreclosures. Ambrose, Buttimer and Capone (1997) relax this by including costs and benefits of events during the delinquency period. While their approach changes the effective default boundary, and thus default-option value, they still equate the default event with ultimate foreclosure. Most recently, Ambrose and Buttimer (2000) modify the option pricing model to explicitly capture the borrower's option to reinstate the mortgage prior to default. As a result, their model is the first to explicitly model both the default (delinquency) and ultimate default (foreclosure) decision. Empirical research is constrained to modeling foreclosure events until more data is available on rates of use of foreclosure alternatives and their value/severity to borrowers and lenders, respectively.[4]

B. Defining Default-option value

In the purest option-pricing approach to mortgage default, focus is on the gain to the borrower from defaulting: the change in expected present value wealth due to exercising an implicit put option on the mortgage. Borrowers are assumed to have the resources necessary to immediately purchase other houses of equal value (i.e., they have downpayment funds), with no impediments to credit access (i.e., no deterioration of credit rating).

In this context, the general default-option value boundary condition specifies the value of default, $D_{i,t}$, as

$$D_{i,t} = \max\left\lfloor D_{i,t+1}, A_{i,t} - V_{i,t} \right\rfloor \qquad (1.)$$

where $D_{i,t+1}$ represents the present value of delaying default to the next period, $A_{i,t}$ represents the present value of the remaining mortgage payments discounted at the current market interest rate (market value of the mortgage liability), and $V_{i,t}$ is the property value.[5] The boundary condition recognizes that the borrower only defaults when

[4] One work in this area is by Ambrose and Capone (1998). They model the probabilities of four different delinquency outcomes: cure, property sale, mortgage assigned to HUD (purchased and serviced by HUD), or foreclosure. Individual firms in the mortgage industry have performed unpublished and proprietary work in an attempt to improve servicing of delinquent mortgages.

[5] Precisely speaking, the value of default-option, $A_{i,t}$ - $V_{i,t}$, is not observable empirically. However, we do observe the outcome of default exercise, i.e., the default loss severity, $U_{i,t}$ - $V_{i,t}$, where $U_{i,t}$ is the present value of unpaid mortgage balance discounted at the mortgage coupon rate. Since $U_{i,t}$ and $A_{i,t}$ are closely related, and the measurement error between $U_{i,t}$ and $A_{i,t}$ does not depend on the underlying stochastic

the benefits of default today outweigh the expected (present value) benefit of defaulting in the future. However, the mortgage pricing literature has long recognized that examining default in isolation ignores the importance of the prepayment option on borrower behavior. The general prepayment boundary condition specifies the value of prepayment, $C_{i,t}$, as

$$C_{i,t} = \max\left[C_{i,t+1}, A_{i,t} - L_{i,t}\right] \qquad (2.)$$

where $C_{i,t+1}$ is the present value of delayed prepayment, and $L_{i,t}$ is the current loan balance (book value of the mortgage liability) at time t.[6]

In order to make operational the default boundary condition for an empirical analysis, we note that the present value of the mortgage liability equals the mortgage book value plus the prepayment option value.[7]

$$A_{i,t} = L_{i,t} + C_{i,t}\,. \qquad (3.)$$

From this perspective, the value of default has four components: the loan balance, $L_{i,t}$, the property value, $V_{i,t}$, the excess cost of the mortgage (i.e., call or prepayment option value), $C_{i,t}$, and the current-and-due payment on the mortgage, P_i:

$$D_{i,t} = L_{i,t} - V_{i,t} + C_{i,t} + P_i \qquad (4.)$$

where i is the loan/property index and t is the time period.

It is convenient to think of default-option value as a percentage of the original

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process $V_{i,t}$, following Kau et al (1992), we will use $U_{i,t}$ - $V_{i,t}$, as a proxy for the true default value $A_{i,t}$ - $V_{i,t}$, in our empirical analysis..

[6] Equation (2) recognizes that the value of the call option, $C_{i,t}$, is the difference between the present value of future mortgage payments discounted at the current market rate and the same payment stream discounted at the note rate.

[7] Drops in interest rates produce a positive effect on the current debt liability and so $C_{i,t}$ is added to L_t. The discounting period is the expected tenure of the family in the home, which may itself be determined in part

house value. This normalizes all house values to $1, so that we need only look at rates and not dollars.[8] Optimal dollar values will vary across borrowers facing the same economic conditions, whereas optimal rates will not. Dropping the *i* subscript for ease of exposition, and dividing by V_0, we have:

$$
\begin{aligned}
d_t &= \frac{L_t}{V_0} - \frac{V_t}{V_0} + \frac{C_t}{V_0} + \frac{P}{V_0} \\
d_t &= l_t - v_t + c_t + p
\end{aligned}
\qquad (5.)
$$

C. Decision to Default

Equations (1) to (5) specify two necessary conditions for borrowers to optimally exercise their default option. First, the value of contemporaneous default must be greater than the highest present value of exercising the option in the future. Second, it is necessary that $(l_t - v_t + p) \geq 0$, because positive call option values will lead to prepayment rather than default if this condition is not met.[9] We summarize these two necessary conditions for optimal default exercise as:

$$
\begin{aligned}
& l_t - v_t + p \geq 0 \\
& \textit{and} \\
& d_t \geq \max\left(\frac{d_{t+s}}{(1+\delta)^s}\right),\ \forall\ s \in \{1,\ldots,T-t\}
\end{aligned}
\qquad (6.)
$$

by interest rates. We follow the Foster-Van Order (1984) convention of using 40 percent of the remaining mortgage term as expected tenure when calculating call option values in empirical research.

[8] Typically, loss severity on mortgage default is thought of through the lender's perspective and the severity rate is the loss as a percentage of the outstanding loan balance at the time of default. Our interest here is in borrower decision making and so the exact normalization we choose is not important. Normalizing by original house price is convenient because it allows us to think of option value as encompassing three sets of changes after time zero: borrower invested equity (L_t/V_0), total market-generated equity (V_t/V_0), and any excess mortgage burden due to changes in interest rates (C_t/V_0).

[9] One can add minimum equity restrictions before refinancing is permitted, but the general principle is still the same.

where δ is the discount rate and T is the minimum of the term of mortgage or the expected tenure in home.

D. Economic Dynamics at the Default Boundary

To empirically model the default decision, we examine the market dynamics that indicate optimal default timing. Given that local and regional house-price cycles in the U.S. average about 10 years in length, and standard mortgage loans are fully amortizing, we can, without loss of generality, think of each mortgage as experiencing one house-price cycle of importance for default decisions.[10]

The second default boundary condition in (6) implies that the borrower will default when the current default value is greater than the present value of next periods default value. If we assume that interest rates are constant ($\partial r / \partial t = 0$), then only changes in property values can result in $d_{t+1} > d_t$. Assuming a smooth, continuous, stochastic house-price process and conditional on having negative equity, the boundary condition suggests that borrowers should default once the rate of increase in the default-option value, d_t, falls to where it equals the discount rate. Thus, the value of delaying default is dominated by the payoff from current default and the borrower exercises the default option.

Three facts regarding default stand out. First, borrowers will exercise the default option when house prices have declined such that the borrower has negative equity. Second, conditional on having negative equity and assuming a similar decline in house

[10] Because of loan amortization, a second house price cycle would not have the same impact on mortgage defaults as would the first cycle. Even if a borrower purchased a home at the peak of a cycle, so that two complete recessions and troughs could be experiences in 15 years, discounting the value of the second

prices, borrowers with older mortgages will exercise the default option prior to borrowers with newer mortgages because delays are more costly in terms of loan amortization. Third, conditional on having negative equity, borrowers will exercise the default option when expected house price movements are minimal.

Now we add interest rate dynamics back into the model, to see how they affect optimal default timing. The benefits from delay now include any projected change in the excess cost of the mortgage. This can have one of two affects on our previous analysis. First, delaying default could dominate current default exercise even when property values have declined if interest rates are falling. Likewise, current default exercise will dominate even for small property value declines if interest rates are rising. Second, the magnitude of a recession (or house price decline) needed to stimulate current default exercise changes as interest rates change: rises in rates require larger recessions to justify default (that is, to achieve $d_t > 0$), and reductions in rates require smaller recessions. It is important to remember that these are marginal effects: large call option values, by themselves, will induce prepayment rather than default (see equation (6), condition 1).

IV. Empirical Testing: Variable Formulation

We can separate the effects discussed above into those that affect default timing, and those that affect default-option value. Once we identify measurable variables that capture these effects, we develop a statistical test of the options-valuation approach to mortgage default. In this section we identify the variables, and in section V we discuss

trough by an additional 10 years would make default optimal during the first trough, even if the second trough were more severe.

data and statistical techniques. Variables discussed here are defined more precisely in Table 1.

A. Default Timing

Optimal default timing can be defined through the relationships shown in equations (6). Our challenge now is to move from that theory to measurable variables. In particular, the first condition of equation (6) says we must measure whether or not default is actually in-the-money at the time of observation. Because we do not have time series of individual property values, we use the Deng, Quigley, and Van Order (1996) measure of the probability that any given property value is below the mortgage balance, at each observation (*PNEQ*).[11] This is a useful construct when analyzing large samples of data from the same geographic area.

Stage of House Price Cycle

The second condition of equation (6) says that we must know whether house price changes are slowing. We capture the stages of the market cycle through use of a categorical variable, *CYCLEn*. We also interact cycle categories with the *PNEQ* variable to see if a greater proportion of borrowers with in-the-money default options actually default during the financially optimal cycle stage, as defined by options theory. We saw earlier that the optimal stage of the cycle for exercising a put option is just before the trough.

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[11] Using the variance of a price index (ε^2), the probability of negative equity, *pneq*, is calculated as

$$PNEQ = \Phi\left(\frac{\log(L)-\log(A)}{\sqrt{\varepsilon^2}}\right),$$

where Φ is the cumulative normal density function. (See Deng (1997), and Deng, Quigley and Van Order (1996)).

Call Option Value

The remaining influence on default timing from equation (6) is the call option value. The second condition of equation (6), indicates that optimal default timing may be accelerated to the extent current market interest rates are below the mortgage coupon rate. Once again, the important issue is how quickly borrowers exercise in-the-money options: when is the strike price reached? Our discussion of equation (4) suggests this will also be a function of movements in interest rates: if they are falling, default may be delayed, whereas if they are rising, default may be accelerated. To address interest-rate effects, we create another categorical variable to interact with *PNEQ*. This one, *RATEn*, captures the difference (spread) between current mortgage coupon rates and the existing contract rate. There should be no significant effect when current rates are near the contract rate, and larger effects as the rate spread increases, in either direction.[12]

Institutional Factors

Timing of default can also be influenced by institutional variables. We include a variable, *DEFJUD*, to indicate the ease in which lenders can obtain deficiency judgments against defaulted borrowers. In States where deficiency judgments are routine, it is more difficult for borrowers to capture the value of the put option, and so optimal default should be exercised less often, and by less risk-averse borrowers.[13] We use this variable

[12] Ideally, we would also like to include variables that indicate the current direction of change in interest rates. However, the lack of any smooth, continuous interest rate patterns makes this problematic. Yet because optimal default should be delayed as interest rates are falling, we can test our theory through use of categorical *RATEn* variables. Categories representing larger declines in interest rates should have higher current-default weights/coefficients than should categories representing smaller declines in interest rates. Thus, the effects of both level and direction of change in interest rates are captured in the categorical variables, which we interact with *PNEQ*.

[13] This issue has been addressed by Jones (1993), Crawford and Rosenblatt (1995), and Ambrose, et al (1997).

even though FHA has a policy of not pursuing defaulted borrowers. Borrowers may not know, *a priori*, of this policy, and lenders may use the potential for court judgments to intimidate borrowers into reinstatement. Because deficiency judgment threats directly affect the proportion of in-the-money default options that will be exercised, we interact the *DEFJUD* with *PNEQ*.

Underlying Mobility Patterns

Borrowers may not always have the luxury of optimal default timing, so we include the mortgage age (*AGE*) to capture the underlying mobility patterns of households. To the extent that mobility forces less-than-optimal default-option exercise, *AGE* coefficients will be statistically significant in the default equation.[14]

B. Default-option value

Our second test of options theory involves default-option value, as specified in equation (5). Do observed, exercised default-option values on foreclosed properties vary as options theory tells us they should? We develop a multivariate regression that provides testable hypotheses for this question.

Default-option value

We confine this analysis to a modified default-option value that does not include the call option value (c_t): [15]

$$\hat{d}_t = l_t - v_t \tag{7.}$$

We could construct a call option value, c_t, and use the full default-option value, d_t, but

[14] Because our sample time frame is only 6 years, we are content with a linear age variable, capturing increasing mobility hazards during the early years of mortgage life.

constructing c_t from market interest rate data would endogenize the call option effect. Therefore, we settle for what we can learn from the impact of interest rates on $\hat{d}_t$. In our calculation of $\hat{d}_t$, loan balance, l_t, is measured using the final outstanding loan balance, and v_t uses the sale price of the foreclosed property less the value of any repairs made on the property prior to sale. Our measure of v_t does not include expenses incurred by the lender either to complete the foreclosure or to manage or sell the resulting property as they are not part of the borrower's default decision. To the extent these lenders expenses influence borrower behavior, it will be through deficiency judgments. We capture such possibilities in a separate variable.

Change in House Prices

Property value, v_t, is a function of market and property factors. Areas with higher average house price growth should have smaller optimal default-option values than should areas with lower average house price growth. House-price-growth volatility is also important for understanding differences in optimal default-option values across borrowers/properties. This is because defaulting borrowers should typically come from the bottom tail of any local house-price distribution. The wider the distribution, the larger will be the potential drop in individual property value and the larger will be the optimal default-option value. So we use two variables to capture differences in optimal default-option values due to property value effects: a local house price index (HPI_t), and its associated volatility measure (VOL_t). The volatility measure is simply the standard deviation of differences in cumulative growth rates at a point in time, and is derived when

[15] This same modified variable was used by Lekkas, Quigley and Van Order (1993), and by Kau et al (1997).

price indexes are computed.[16]

Classification Variables

Loan balance over time (l_t), reflects borrower investment in the property through both initial downpayment and loan amortization. Borrowers with greater investments will have smaller optimal default-option values for any given set of economic circumstances. We capture borrower investment in two variables. The first is a categorical variable for initial loan-to-value ratio, *LTVnn*. We use this rather than a continuous measure of LTV to follow the practice of mortgage underwriters and insurers in classifying mortgage risk. Optimal default values—the value a borrower can extract through exercising the put option—will be directly related to LTV. The second investment variable is an age variable used to proxy for differences in loan amortization. We do not include amortization directly because all of the loans are newly originated and in the first 5 years of life where actual amortization is fairly constant over time. However, because differences in amortization should affect default-option values, we include the time/age variable to eliminate any systematic variation in option value due to amortization over time.

A second classification variable used in stage two is a categorical variable for initial loan amount (*LOANnn*). This variable provides an indication of any impact that *dollar* default-option values might have on optimal, exercised default-option value ratios.

Call Option Value

Our theory tells us that changing interest rates will cause optimal default to occur

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[16] We actually calculate these standard deviations from parameters provided with the house price index data. These parameters are derived as part of the price-index generation process.

at a time when $\hat{d}_t$ is less than its maximum value. When call option values are large, defaults might occur faster, and thus exercised default-option values, $\hat{d}_t$, would be smaller. So we use call option value, $CALL_t$, as an explanatory variable in our default-option value regression.[17] To make the call value variable appropriately scaled for each borrower, and scaled to match $\hat{d}_t$, we divide $CALL_t$ by the original house price, V_0, to get the scaled variable, c_t.

Institutional Constraints

Borrowers will not realize the full value of d_t (or $\hat{d}_t$) if the lender obtains deficiency judgments against them. However, deficiency judgments do not change the value of the default option as we measure it, just the proportion of the default-option value that borrowers retain. Still, deficiency judgments include the foreclosure costs of the lender, which are not a part of the default-option value to the borrower, so they can turn positive option values into negative values. Risk-averse borrowers in States with readily available (for lenders) deficiency judgments may only exercise their put options at higher potential payoffs from default. We add the fixed effect variable, *DEFJUD*, for States in which deficiency judgments are difficult to obtain in order to test for any (negative) effect on exercised default-option value.[18]

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[17] Following Foster and Van Order (1984), $CALL_t$ is calculated as the present value of the mortgage at the current market interest rate (assuming the borrower prepays after 40% of the remaining term has passed) less the current mortgage balance.

[18] States do not generally outlaw deficiency judgments outright, but rather place restrictions on how and when they can be used.

Self-Selection Correction

Finally, we follow the LQV study by including an inverse Mills-ratio borrower self-selection correction factor (*MILLS*$_t$). This factor corrects for the fact that the population of loan defaults are a nonrandom sample from a larger population of loans that all have an associated default-option value. We compute the inverse Mills ratio from the first-stage mortgage default probability model. The inverse Mill's ratio is defined as $\lambda(x'\beta) = \phi(-x'\beta)/(1-\Phi(-x'\beta)) = \phi(x'\beta)/\Phi(x'\beta)$, if default, and $-\lambda(-x'\beta) = -\phi(-x'\beta)/\Phi(-x'\beta)$, if not default, where $\phi(\cdot)$ and $\Phi(\cdot)$ are pdf and CDF function, respectively. $\lambda(\cdot)$ is also known as the hazard rate. Heckman (1976) has shown that by including this inverse Mill's ratio into the second stage OLS regression, it corrects the sample selection bias, and provides a consistent estimator.

C. Summary of Variables

The calculation of each explanatory variable is detailed in Table 1. To summarize the discussion above, the first-stage model of default probability includes age of the loan, the probability of negative equity (*PNEQ)*, house price cycle stage (*CYCLEn*), the interaction of *PNEQ* with the *CYCLEn* variables and interest-rate spread categories (*RATEn*), and an interaction of *PNEQ* with ease of lenders obtaining deficiency judgments in each property State (*DEFJUD*).

Among the cycle-stage variables, *CYCLE1* recognizes periods when local house prices have entered a true downturn, as measured by a drop in house prices of at least 5 percent. *CYCLE2* represents the bottom segment of a cyclical downturn, where property values have dropped at least 10 percent. *CYCLE3* recognizes the trough and initial recovery phase of a price cycle when, after crossing through *CYCLE2*, prices have stabilized and started to rise. See Table 1 for exact calculations of the *CYCLEn* variables.

We categorize the spread between current mortgage rates and the contract rate into 5 groups (*RATEn*), where *RATE3* represents stable rates and in used as the baseline. Other *RATEn* variables provide an indication of the extent to which the prepayment option is in- or out-of-the-money.

The stage two model of default severity includes variables that measure the movement and volatility of house prices (*HPI*, *VOL)*, the call option value (*c*), the LTV category (*LTVn*), the original loan amount category (*LOANn*), the presence of state laws restricting deficiency judgments (*DEFJUD*), and the inverse Mills ratio (*MILLS*).

V. Statistical Analysis

A. Data Sources

Mortgage data used for this study come from historical records on 30-year fixed-rate mortgages on single-family, non-investor mortgages insured by FHA and originated in 1989. We use just one origination year because our longitudinal database for stage one gets too large for multiple loan cohorts. After matching and eliminating records with missing data, we have a dataset of 116,415 loans and 6.2 million observations. Of these loans, 4,306 (3.7 percent) defaulted during the observation period, which ends with December 1995.[19]

In order to calculate default value, we match loan defaults with post-default records to identify post-foreclosure property sales prices. Unfortunately, our default-option value sample is reduced by a large number of loans either assigned to HUD—rather than foreclosure and property sale—or whose underlying properties were sold to local government agencies and non-profit agencies at negligible prices. Thus, for the default-option-value analysis, we screen out all defaults with missing or artificially low sales prices (prices less than 10% of the loan's unpaid balance). This leaves a sample of 2,516 defaults with sufficient data for option-value analysis.

House price index data by city (MSA) comes from Freddie Mac and from the Office of Federal Housing Enterprise Oversight (OFHEO). Each computes a weighted repeat sales price index based upon Fannie Mae and Freddie Mac repeat transactions data, on a quarterly basis.[20] But only OFHEO makes available the additional parameters necessary to calculate the *PNEQ* variable. The mortgage interest rate series used here is the historical FHA 30-year fixed rate mortgage series, which is reported on a monthly basis and published in the *Federal Reserve Bulletin*.

Table 2 shows descriptive statistics for the variables used in the first- and second-stage regressions. For stage one, mean values are across all loans and monthly observations. During the sample period, the mean probability of negative equity was 36 percent, with a range of between 0 and 79 percent. Very few observations actually occurred during housing cycle downturns. Just 1.9 percent of the observations were in an initial period of a house-price-cycle decline (*CYCLE1*), 1.0 percent in the depths of a decline (*CYCLE2*), and 0.9 percent were in a cyclical trough (*CYCLE3*). This is an unfortunate result of the limits of our study period (1989-95). Major housing cycle

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[19] Our FHA termination records end in early 1996.
[20] OFHEO indices are available via its web site, www.ofheo.gov.

downturns occurred in many MSAs in the mid- to late-1980s (southwest and northeast), and again in 1995-96 (California).

The bottom half of Table 2 shows descriptive statistics on variables in the second-stage, default-option-value analysis. The mean observed default-option value is 17 percent, with a range from –50 percent to near 90 percent. This mean value appears smaller than what is reported in other studies (over 30 percent), but that is because this is pure borrower default-option value and not loss severity to the lender. Table 3 provides a full frequency distribution of exercised default option values. The median value is 0.13 and The inner-quartile range (25^{th} to 75^{th} percentiles) is from 0.02 to 0.30. We will discuss exercised option values more after discussion the regression results.

B. First-Stage Regression Results – Default Timing

Baseline Default Variables

We report default-timing coefficient estimates in Table 4. Positive coefficients on *AGE* and *PNEQ* indicate standard results, that the probability of default increases both as the mortgage ages (through year 5 in our sample) and as the potential for negative equity increases. The *CYCLEn* variables, by themselves, indicate that the baseline hazard rates of default increase significantly as loans enter and move toward the trough of house price cycles.

Interactions with PNEQ

The interactive variables are quite interesting. The interaction of *PNEQ* with the *CYCLEn* categories shows how the potential size of the default-option value becomes less meaningful in helping to predict default as loans enter market downturns. This is as theory predicts: At the optimal default timing (*CYCLE2*) all in-the-money options should be exercised, regardless of their values relative to one another. Thus loans with smaller and larger rates of *PNEQ* may default at the same rates, so long as default is in-the-money. The Interaction of *PNEQ* with *CYCLE2* has the largest (negative) effect, more than fully offsetting the base effect of *PNEQ* by itself, and thus reversing the interpretation of *PNEQ* when optimal default timing exists. The larger size of the interaction of *PNEQ* with *CYCLE3* versus that with *CYCLE1* is also instructive. It shows that borrowers seeking to optimally default are more likely to wait until they see the actual bottom of the market, rather than to default as soon as default is in the money. This provides some confirmation of the value of delayed default imbedded in mortgage (option) pricing models, though pricing models have relied upon random house price movements rather than simulated price cycles.

The interactions of *PNEQ* with the *RATEn* confirm theoretical expectations that spreads between current interest rates and contract rates do matter. These results also confirm that it is proper to look at the market value of the debt liability (including call option value), rather than just book value, when evaluating default options. We cannot, however, speak decisively regarding the marginal value of large increases or declines in interest rates (over 200 basis points, *RATE1* and *RATE5*). The most important distinction we find is between *PNEQ•RATE2* and *PNEQ•RATE4*, where interest rate shifts are between 100 and 200 basis points up or down. There are very few observations of *RATE1*, and the *PNEQ•RATE1* interaction does not produce a statistically significant effect.

In States where it is relatively difficult for lenders to obtain deficiency judgments, we find that *PNEQ* becomes more important: each percentage increase in the potential for option value increases default rates more for borrowers who do not fear deficiency judgments more than it does for borrowers who might fear deficiency judgments (*PNEQ•DEFJUD > 0).*

C. Second-Stage Regression Results—Default-option value

We use ordinary least squares regression to model default-option values. As mentioned earlier, default-option value is measured exclusive of the call-option value. Regression results are reported in Table 5. We find evidence that variables affecting optimal default boundaries do influence observed, exercised default-option values. Yet the overall regression fit is sufficiently low (R^2=0.18) that we cannot rule out there being significant unobservable influences on optimal default timing and value. Thus we conclude that put-option exercise can be considered one influence on default incidence, but clearly not the only criteria. Table 6 provides a matrix of predicted option values by LTV and loan size class, using mean values of other variables. (Table 3 shows the distribution of exercised default option values found in the stage two regression sample.)

LTV and Loan Size

Because optimal default timing is a function of house price cycles, and not loan characteristics *per se*, we expected to find that lower loan-to-value categories have smaller observed default-option values. This is found in the regression results and seen in Table 5, but the distinctions across LTV class are not as marked as would be expected if borrowers are defaulting in optimizing fashion. Indeed, there is no measurable distinction in LTV class effects until we get to the lowest (*LTV70*) class. Lekkas, et al (1993), using conventional market foreclosure data, find much stronger differences by LTV class, whereas ours are captured more in HPI and loan size class.[21]

Default option values increase as one moves away from the *LOAN100* class ($75,000-$100,000). In 1989, FHA insured mortgages up to 95 percent of area median house prices, and the upper limit on insurable loans was $105,000, so the relationship among loan-amount-class variables likely represents differences in house price appreciation rates across house-price classes. The rich middle quartiles will have more stable house prices (around an HPI trend), and likely have better appreciation rates over time than the outlying quartiles. Thus, optimal, exercised default values should vary as seen here across loan classes.

Call Option Value

Our prior expectation on the roll of call option value, *c*, was that increases in this variable would tend toward default being exercised at lower levels of measured default-option value—lower than the optimal level—through effects on default timing. This is not born out by the positive coefficient on *c*. Yet because positive call options, by themselves, induce prepayment, the positive coefficient we find could indicate that when call options are in the money, the default-option value must be even greater to overcome the value of prepayment and lead to default (rather than prepayment) exercise.[22] A one standard deviation increase in call option value (8 percent of original house price) leads to

[21] Our interpretation of the Lekkas, *et al,* findings is different from theirs. Based on mortgage valuation equations, they believed that higher LTV loans should have lower exercised default-option values. However, we show in this paper that what matters for optimal default exercise is what happens to bring option value across the boundary conditions, which is where decisions are made. Thus, we would claim that the Lekkas, *et al* results do confirm options theory.

[22] The first condition in equation (6) indicates that observed default-option value—without the call option—must first be in the money before default will be considered a viable option.

an average 1.9 percent increase in exercised default-option value.

House Price Effects

The coefficient on *HPI* tells us that each 1 percentage point increase in area house prices will result in a smaller, average 0.58 percentage point drop in exercised default-option values. Given that this result is independent of changes in *VOL*, it must be that average appreciation rates on properties whose loans end in foreclosure are somewhat less than average appreciation in a metropolitan area. This makes sense, because homes in pockets with less-than-average appreciation rates will be more likely to have in-the-money default options.

D. Simulations on Second Stage Results

To provide a better understanding of the magnitude of default option values over the house-price cycle, we use our stage-two regression results to simulate expected values by LTV and loan size categories, as *HPI* increases from 0.90 (10 percent decline) to 1.50. All other variables are held at sample mean values. Predictions in Figure 1 fix loan size to the 75-100 class (*LOAN100*), and show outcomes by LTV class, while predictions in Figure 2 fix LTV to the above 90 percent class (*LTV95*) and show outcomes by loan-size class. Because the model is linear in parameters and variables, the LTV class results in Figure 1 are parallel to one another, and likewise for the loan size results in Figure 2.

In each Figure, the point where the option-value lines cross the zero line on the vertical axis indicates the HPI levels at which expected default values are zero ($\hat{d}_t = l_t - v_t = 0$). In Figure 1, the zero-value HPI levels by LTV class are 1.025 (*LTV70*), 1.15 (*LTV80*), 1.175 (*LTV90*), and 1.20 (*LTV95*). This highlights how borrowers that default have properties in subsectors of housing markets that have less than average price appreciation. For example, the zero-value HPI level of 1.20 for *LTV95* means that loans that go to default in MSAs with 20 percent price appreciation (since loan origination), will have instead had about 5~10 percent price depreciation.[23] The zero-value HPI level of 1.15 for loans in the *LTV80* class suggests that loans defaulting in areas with 15 percent total appreciation have an average property value depreciation of 20~30 percent, depending on actual initial LTV.

Table 7 provides information on the distribution of HPI values for defaulting loans by LTV class. While the mean MSA-level HPI increases slightly with LTV class, the maximum HPI increases much faster.[24]

Figure 2 looks at how exercised default option values fall with HPI across loan-size classes, for loans in the LTV95 category. Because the 75-100 loan size class generally represents mid-level house prices, loans in this class will have the most stable house price appreciation relative to area means. Here we see that the zero-value HPI level is 1.120 for LOAN100, 1.25 for LOAN125, 1.30 for LOAN75 and 1.50 for LOAN50. Houses in the lower price ranges appear to have much worse than average price performance possibilities.

VI. <u>Conclusions</u>

Option-pricing theory teaches that default is exercised when the current value of

[23] The *LTV95* class includes all loans with LTV above 90 percent. The modal LTV is just under 95 percent.
[24] Generally, higher HPI values mean more time has transpired, and so the variance of property-specific price appreciation rates is also larger. The point of Table 7, however, is that as HPI increases, the likelihood that default will be in-the-money diminishes faster for loans of lower LTV classes.

exercise is positive, and is greater than the (present) value of delayed exercise. This creates a boundary or trigger problem that we understand by looking at the first derivatives of default value with respect to underlying factors, namely house prices and interest rates. Our primary research question is, what economic conditions will lead to optimal default exercise? What should be happening with house prices and interest rates to indicate that optimal timing and value are achieved? This question leads us to a two-part empirical model of default exercise and option value, which we test using variables that help us capture rates of default exercise as economic factors change. In particular, our default rate model captures the interaction of probabilities of negative equity with stages of the house-price cycle and interest rate movements.

Our default-option-value model is not especially unique in its structure, but rather in its interpretation. One previous study that looked at borrower option values at default exercise (Lekkas, *et al*, 1993) did not analyze the results from the frame of reference of optimal exercise. This, we believe, led to some erroneous conclusions, especially with respect to the relationship between LTV categories and default option value. Other studies have looked at the resulting loss to the lender, rather than borrower decision making. Lender loss/cost is the exercised borrower option value *plus* costs of foreclosure and property disposition.

Our results generally support an option specification for default modeling, though error terms are large enough that one cannot conclude that all borrowers treat default as a financial option. Most striking in our results are the interaction effects of house-price-cycle-stage with the probability of negative equity. The relationship of the probability variable to default rates breaks down as housing cycles enter significant downturns: when optimal default timing is reached, the absolute level of negative equity no longer matters. All borrowers with in-the-money options should default. This effect has not been captured in previous empirical analysis. Our own modeling could be improved by having a longer data series. Our time frame was 1989-1995, a time that just misses major cyclical troughs (20 percent price declines) in several MSA markets.

Could heterogeneous transaction costs account for the remainder of the variation in observed default values? Likely not, given the large positive equity values found in some defaults (see Table 5). Loan servicers indicate that one of the most common causes of default is divorce. When such family disruptions are accompanied by personal acrimony, even property equity can become a casualty. Yet transaction costs of home sale do help explain how borrowers might default when the observed option value is close to or even above zero. Such default is generally considered "suboptimal" in that there must be some trigger event like a relocation need that forces mortgage termination at a time other than when the underlying financial options are optimally exercised.

References

Ambrose, Brent W., and Richard J. Buttimer. 2000. “Embedded Options in the Mortgage Contract,” *Journal of Real Estate Finance and Economics* 21:2 (Forthcoming).

Ambrose, Brent W., Richard J. Buttimer, and Charles A. Capone, Jr. 1997. “Pricing Mortgage Default and Foreclosure Delay,” *Journal of Money, Credit, and Banking* 29 (3, August), 314-325.

Ambrose, Brent W. and Charles A. Capone, Jr. 1996. "Cost-Benefit Analysis of Single-Family Foreclosure Alternatives,” *Journal of Real Estate Finance and Economics* 13, 105-120.

________. 1998. “Modeling the Conditional Probability of Foreclosure in the context of Single-Family Mortgage Default Resolutions,” *Real Estate Economics* 26 (3, Fall), 391-430.

Berkovec, James A., Glenn B. Canner, Stuart A. Gabriel, and Timothy Hannan .1998. “Discrimination, Competition, and Loan Performance in FHA Mortgage Lending,” *Review of Economics and Statistics* 80 (2, May), 241-50.

Capone, Jr., Charles A. 1996. *Providing Alternatives to Mortgage Foreclosure: A Report to Congress*. Washington, DC: U.S. Department of Housing and Urban Development, August (HUD-1611-PDR).

Clauretie, Terrence. 1990. “A Note on Mortgage Risk: Default vs. Loss Rates,” *AREUEA Journal* 18 (2), 202-206.

Crawford, Gordon, and Eric Rosenblatt. 1995. “Efficient Mortgage Default Option Exercise: Evidence from Loss Severity,” *Journal of Real Estate Research* 19 (5), 543-555.

Deng, Yongheng. 1997. “Mortgage Termination: An Empirical Hazard Model with a Stochastic Term Structure,” *Journal of Real Estate Finance and Economics*, 14 (3), 309-331.

Deng, Yongheng, John M. Quigley, and Robert Van Order. 1996. “Mortgage Default and Low Down-payment Loans: The Cost of Public Subsidy,” *Regional Science and Urban Economics* 26, 263-285.

Deng, Yongheng, John M. Quigley, and Robert Van Order. 2000. “Mortgage Terminations, Heterogeneity and the Exercise of Mortgage Options,” *Econometrica* 68 (2), 275-307.

Elmer, Peter J. 1997. *A Choice Theoretic Model of Mortgage Default*, unpublished manuscript. Washington, DC: Federal Deposit Insurance Corporation.

Foster, Chester, and Robert Van Order. 1984. “An Option-Based Model of Mortgage Default,” *Housing Finance Review* 3, 351-372.

Heckman, James. 1979. “Sample Selectivity Problems as a Specification Error,” *Econometrica* 44, 153-161.

Jones, Lawrence. 1993. “Deficiency Judgments and the Exercise of the Default Option in Home Mortgage Loans,” *Journal of Law and Economics* 36, 115-138.

Kau, James B., Donald C. Keenan, Walter J. Muller III, and James F. Epperson. 1992. “A Generalized Valuation Model for Fixed-Rate Residential Mortgages,” *Journal of Money, Credit, and Banking* 24(3), 279-99.

Kau, James B. and Donald C. Keenan. 1993. “Transaction Costs, Suboptimal Termination, and Default Probabilities for Mortgages,” *AREUEA Journal* 21(3), 247-63.

Kau, James, and Donald C. Keenan. 1999. "Patterns of Rational Default," *Regional Science and Urban Economics*, 29, 217-244.

Lekkas, Vassilis, John M. Quigley, and Robert Van Order. 1993. “Loan Loss Severity and Optimal Mortgage Default,” *AREUEA Journal* 21(4, Winter), 353-372.

Quigley, John M., and Robert Van Order. 1995. “Explicit Tests of Contingent Claims Models,” *Journal of Real Estate Finance and Economics* 11, 99-117.

Titman, S., and W. N. Torous. 1989. "Valuing Commercial Mortgage: An Empirical Investigation of the Contingent Claims Approach to Pricing Risky Debt," *The Journal of Finance*, 44, 345-373.

Vandell, Kerry. 1995. “How Ruthless is Mortgage Default?” *Journal of Housing Research* 6 (2), 245-264.

Table 1
Variables Used in Regressions

Variable name	Variable label	Description
Stage One: Termination Rates		
Probability of negative equity	*PNEQ*	Probability that a given loan will have an in-the-money default option in a given observation period.
Stage of housing market cycle	*CYCLEn*	*CYCLE1* = 1, MSA house prices have fallen 5-10 % over the past 24 or 36 months. *CYCLE2* = 1, MSA house prices have fallen 10% or more over the past 24 or 36 months. *CYCLE3* = 1, MSA house prices have passed through CYCLE 2 and have risen over the past 12 months, but total increase over the past 36 months is less than 5 percent.
Interest rate spread r_0=contract rate r_t=market rate	*RATEn*	*RATE1* = 1, $(r_0 + .02) \leq r_t$ *RATE2* = 1, $(r_0 + .01) \leq r_t < (r_0 + .02)$ *RATE3* = 1, $(r_0 - .01) < r_t < (r_0 + .01)$ *RATE4* = 1, $(r_0 - .01) < r_t < (r_0 - .02)$ *RATE5* = $r_t \leq (r_0 - .02)$
Deficiency judgments	*DEFJUD*	*DEFJUD* =1, difficult in property state to obtain a deficiency judgment against defaulting borrower (0, otherwise)
Mortgage age	*AGE*	Log(months) = age since loan origination. (Sample includes first 5-6 years of loan life.)
Stage 2: Default-option value		
Default-option value (observed)	$\hat{d}_{it}$	Defaulting loan balance plus repairs less actual property sale price
House Price Index value	$HPI_{i,t}$	MSA-level House Price Index = 1 plus cumulative house price growth in property MSA from time of loan origination to default.
House Price Index volatility	*VOL*	Standard deviation of the $HPI_{i,t}$, computed based on volatility parameters of the MSA-level price index series.
Call option value	*c*	Market value of mortgage liability less book value, divided by original house price
Original LTV class	*LTV60* *LTV70* *LTV80* *LTV90* *LTV95*	dummy variable, loans with $0<LTV\leq 60$ dummy variable, loans with $60<LTV\leq 70$ dummy variable, loans with $70<LTV\leq 80$ dummy variable, loans with $80<LTV\leq 90$ dummy variable, loans with $90<LTV$
Deficiency judgments	DEFJUD	dummy variable for States with restrictions on deficiency judgments (same variable used in Stage 1)
original mortgage amount	*LOAN50* *LOAN75* *LOAN100* *LOAN125*	original loan amount categorical variables: LOAN50: under $50,000 LOAN75: $50-75,000 LOAN100: $75-100,000 LOAN125: $100-125,000
inverse Mills ratio	$MILLS_{it}$	Selection-bias correction factor from stage one.

Table 2

Descriptive Statistics on Variables used in Statistical Analysis

Variable	Mean	Standard dev.	Minimum	Maximum
Stage 1: Probability of Default Exercise				
PNEQ	0.361	0.111	0.000	0.788
CYCLE1	0.019	0.135	0	1
CYCLE2	0.010	0.100	0	1
CYCLE3	0.009	0.094	0	1
RATE1	0.046	0.209	0	1
RATE2	0.228	0.419	0	1
RATE3	0.647	0.478	0	1
RATE4	0.070	0.255	0	1
RATE5	0.009	0.096	0	1
DEFJUD	0.285	0.452	0	1
Ln(AGE)	3.093	0.944	0.000	4.431
observations	6,195,216			
Stage 2: Default Option Value				
$\hat{d}_{it}$	0.172	0.222	-0.502	0.897
$HPI_{i,t}$	1.042	0.052	0.898	1.406
VOL	0.112	0.034	0.023	0.232
c	-0.011	0.080	-0.234	0.301
LTV70	0.015	0.122	0.000	1.000
LTV80	0.066	0.248	0.000	1.000
LTV90	0.112	0.315	0.000	1.000
LTV95	0.806	0.396	0.000	1.000
DEFJUD	0.310	0.463	0.000	1.000
LOAN50	0.279	0.448	0.000	1.000
LOAN75	0.367	0.482	0.000	1.000
LOAN100	0.274	0.446	0.000	1.000
LOAN125	0.080	--	0.000	1.000
MILLS	7.158	0.522	5.876	10.061
observations	2,515			

Table 3
Distribution of Exercised Default Option Values
in Data Sample[a]

Value range	Frequency	Percent	Cumulative Percent
$-0.6 \le \hat{d}_{it} < -0.5$	2	0.1	0.1
$-0.5 \le \hat{d}_{it} < -0.4$	3	0.1	0.2
$-0.4 \le \hat{d}_{it} < -0.3$	21	0.8	1
$-0.3 \le \hat{d}_{it} < -0.2$	84	3.3	4.4
$-0.2 \le \hat{d}_{it} < -0.1$	224	8.9	13.3
$-0.1 \le \hat{d}_{it} < 0.0$	191	7.6	20.9
$0.0 \le \hat{d}_{it} < 0.1$	272	10.8	31.7
$0.1 \le \hat{d}_{it} < 0.2$	540	21.5	53.1
$0.2 \le \hat{d}_{it} < 0.3$	403	16.0	69.2
$0.3 \le \hat{d}_{it} < 0.4$	266	10.6	79.7
$0.4 \le \hat{d}_{it} < 0.5$	207	8.2	88
$0.5 \le \hat{d}_{it} < 0.6$	125	5.0	92.9
$0.6 \le \hat{d}_{it} < 0.7$	86	3.4	96.3
$0.7 \le \hat{d}_{it} < 0.8$	58	2.3	98.6
$0.8 \le \hat{d}_{it} < 0.9$	25	1.0	99.6
$0.9 \le \hat{d}_{it}$	9	0.4	100

[a] Default option values are measured as percentages of original house values.

Table 4
First Stage Probit Regression of Probability of Default Exercise
Results

Variable	Coefficient estimate	Standard error	χ^2 Statistic	probability Value
CONSTANT	-10.417	0.119	7609	0.000
Ln(AGE)	0.282	0.019	221.4	0.000
PNEQ	5.964	0.202	869.4	0.000
*PNEQ*CYCLE1*	-3.537	0.673	27.62	0.000
*PNEQ*CYCLE2*	-8.045	0.905	79.06	0.000
*PNEQ*CYCLE3*	-4.815	1.558	9.56	0.002
CYCLE1	1.927	0.287	45.00	0.000
CYCLE2	3.129	0.395	62.64	0.000
CYCLE3	1.877	0.760	6.09	0.136
*PNEQ*RATE1*	-0.305	0.189	2.59	0.107
*PNEQ*RATE2*	-0.684	0.096	50.99	0.000
*PNEQ*RATE4*	0.729	0.122	35.57	0.000
*PNEQ*RATE5*	0.982	0.316	9.66	0.002
*PNEQ*DEFJUD*	0.665	0.080	69.39	0.000
Log-Likelihood	-37603		75206	0.000
observations	6,195,216			
Defaults	4,681			

Table 5

Second Stage (Exercised) Default Option Value Regression Results

Variable	Coefficient estimate	Standard error[a]	t-Statistic	Probability Value
INTERCEP	1.145	0.058	19.86	0.000
AGE	0.001	0.0004	2.37	0.018
CALL	0.181	0.045	4.02	0.000
LTV70	-0.105	0.027	-3.92	0.000
LTV80	-0.025	0.014	-1.68	0.093
LTV90	-0.008	0.010	-0.751	0.453
HPI	-0.5822	0.078	-7.49	0.000
VOL	-0.740	0.143	-5.18	0.000
DEFJUD	0.038	0.008	4.96	0.000
LOAN50	0.164	0.012	14.15	0.000
LOAN75	0.047	0.011	3.98	0.000
LOAN100	-0.022	0.011	-2.01	0.045
MILLS	-0.051	0.011	-4.91	0.000
R^2	0.184			

[a] **Consistent asymptotic standard errors using Heckman (1977) adjustments based on first stage regression results.**

Table 6

Predicted Default Option Value
by LTV and Loan Size Categories

Using mean values of all other variables

	LOAN50	*LOAN75*	*LOAN100*	*LOAN125*
LTV70	0.1734	0.0538	-0.0132	0.0091
LTV80	0.2539	0.1343	0.0673	0.0896
LTV90	0.2709	0.1513	0.0843	0.1066
LTV95	0.2784	0.1588	0.0918	0.1141

Table 7

Observed MSA-Level HPI Values at Time of Loan Defaults, by LTV Class

		HPI statistics		
LTV class	Observations	Mean	Minimum	Maximum
LTV ≤ 70	38	1.031	0.921	1.184
70 < LTV ≤ 80	165	1.035	0.908	1.216
80 < LTV ≤ 90	281	1.040	0.898	1.238
90 < LTV	2026	1.043	0.898	1.401

Figure 1

Predicted Default Option Values by LTV Class

For Loan Sizes 75-100

(All other variables values at sample means)

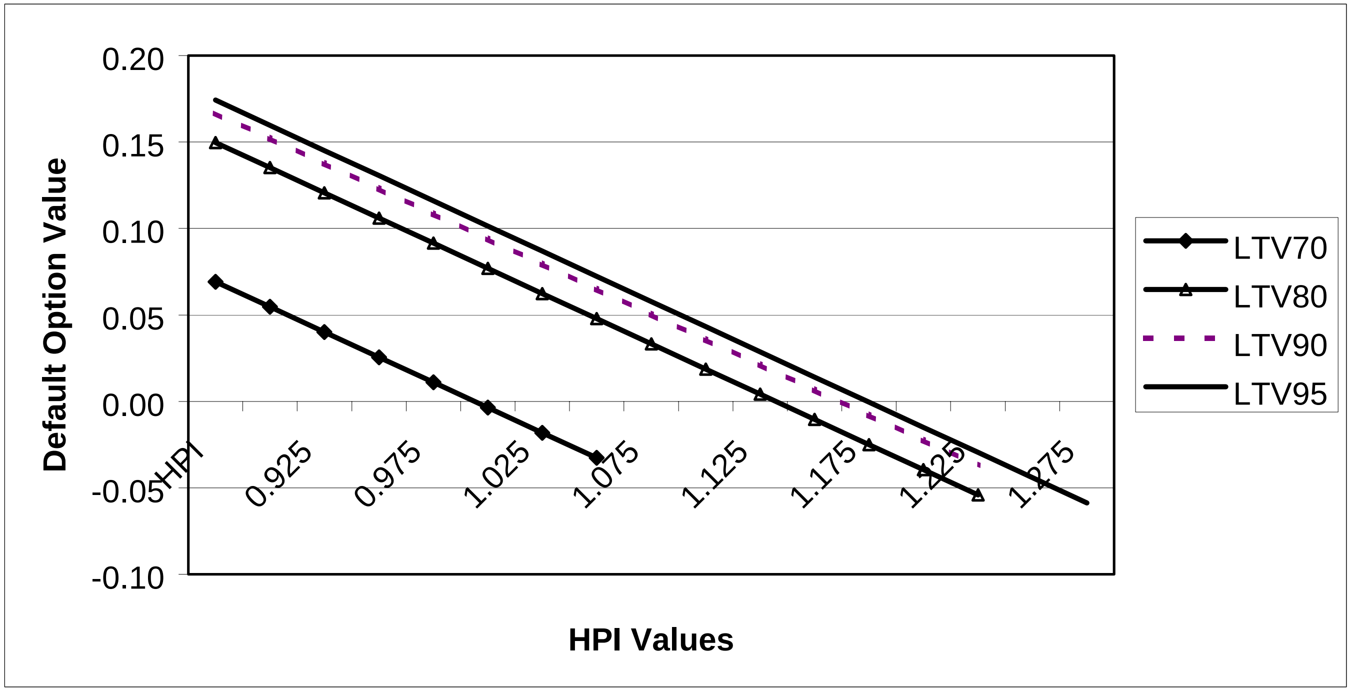

Figure 2

Predicted Default Option Values by Loan Size Class
For LTV95 Class
(All other variables values at sample means)